Analog and Hybrid Computing

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Properties of an analog computer

An (electronic) analog computer

- operates on continuous values.

- has a range of values limited by the so called *machine unit* which is normally 10 V or 100 V.

- is inherently parallel and tops (nearly) every digital computer in this respect (apart from DDAs).

- has a quite limited precision.

- is a perfect match for solving scientific and engineering problems which can be described by differential equations.

- is just wonderful!

Slide rules

The simplest analog computer is the well known slide rule as shown below:
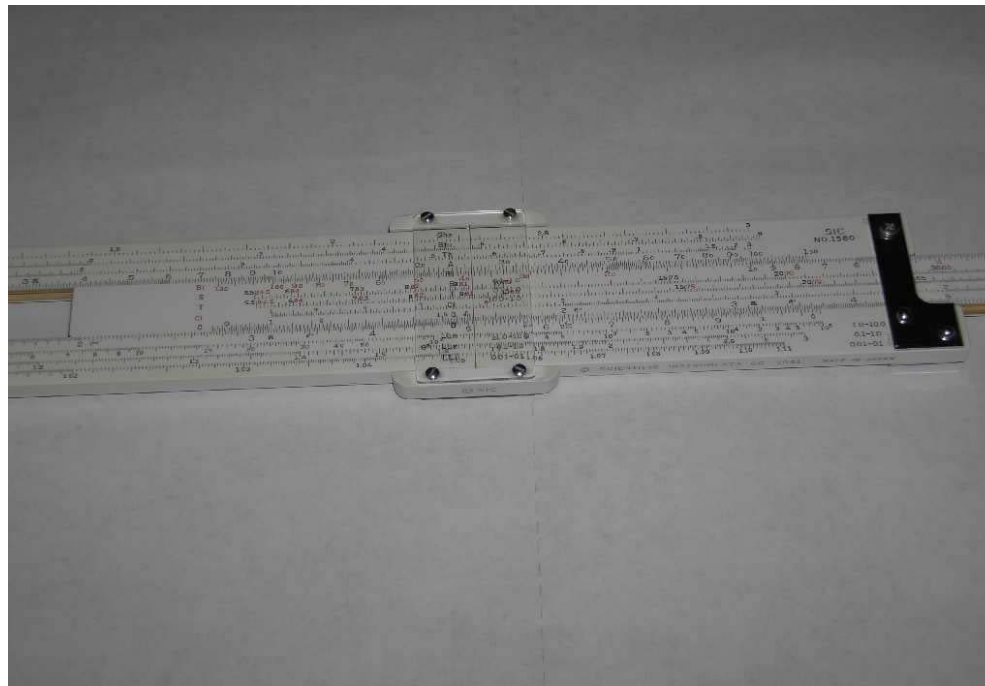
Values are represented by distances which allows easy adding and subtracting of values and thus multiplication and division by using a logarithmic scale.

Modern slide rules

Even today slides rules are used in areas where reliability and ease of use are of prime importance. An example of this, the "E6-B Flight Computer" is shown below:

Planimeters

A so called "planimeter" is capable of calculating the area of a planar region. Such an integrating device is shown below:

Differential Analyzers

Apart from simple devices like these, more complex instruments have been devised by various scientists like James Thomson (the brother of Lord Kelvin) and for example Vannevar Bush.

At the heart of instruments like these were mechanical devices capable of integration, summing and function generation.

The wheel integrator

The most important element of such a differential analyzer is a wheel integrator as shown below (see [9][p.24]):
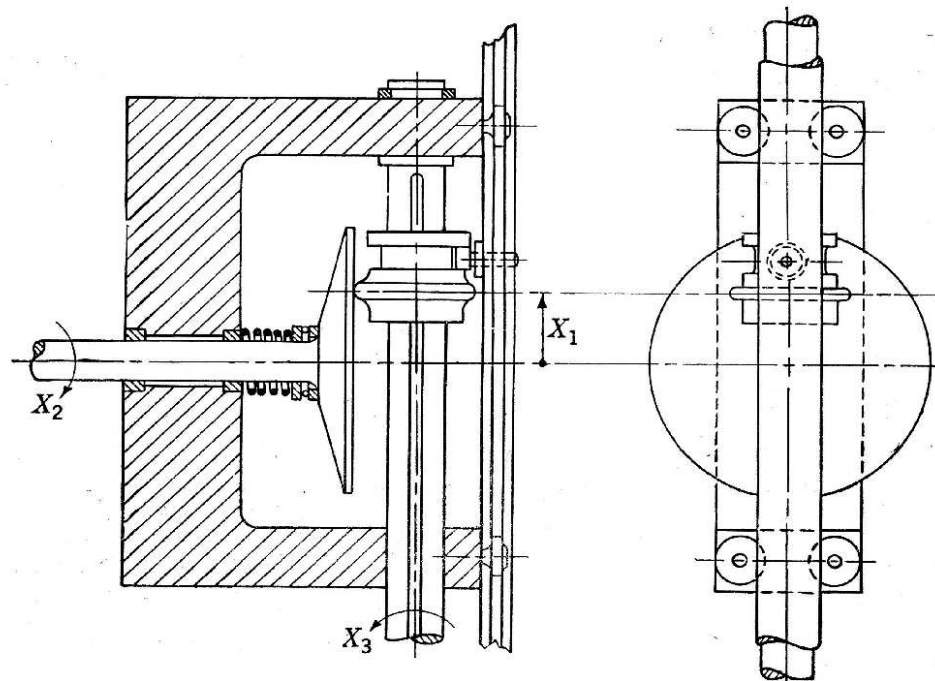
The wheel integrator

A wheel integrator consists of a (glass) disk which is mounted on a rotating shaft. The angular velocity (or position) is the free variable of the integration to be performed (most often the time t).

A (steel) wheel with a sharp edge is mounted on a shaft and is placed on the rotating disk in such a manner that the rotation of the glass disk results in a rotation of the wheel.

The ratio of disk movement and wheel movement depends on the position of the wheel on the disk with respect to the center of the disk. If the wheel is located at the center, it will not rotate at all while it will rotate at maximum speed when its position is on the edge of the disk.

The wheel position is thus the variable to be integrated.

The mechanical differential analyzer

The following picture shows a complete setup of a simple differential analyzer integrating over an arbitrary function (gained from an "input table"). The result of the integration is plotted on an "output table" (see [5][p.190]):
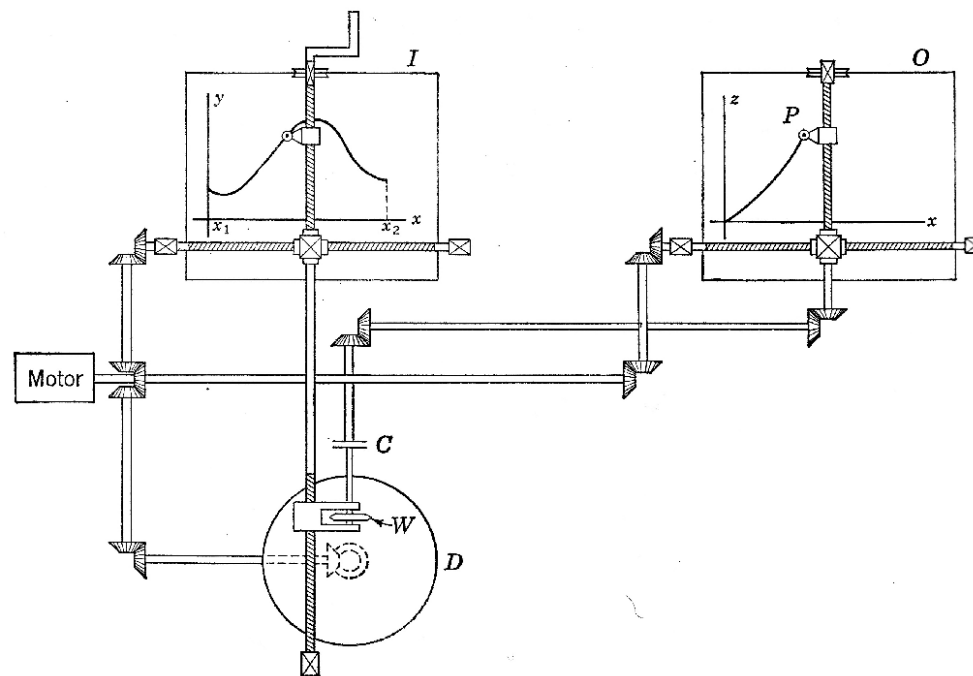
Tim Robinson's mechanical differential analyzer

Tim Robinson constructed a mechanical differential analyzer using Meccano parts only. The picture below shows two of his glass disk integrators (cf. [8]):

Tim Robinson's mechanical differential analyzer

The next picture (cf. [8]) shows the overall setup of this differential analyzer. On the left are the double output table and the input table while the integrators and interconnection shafts can be seen on the right:
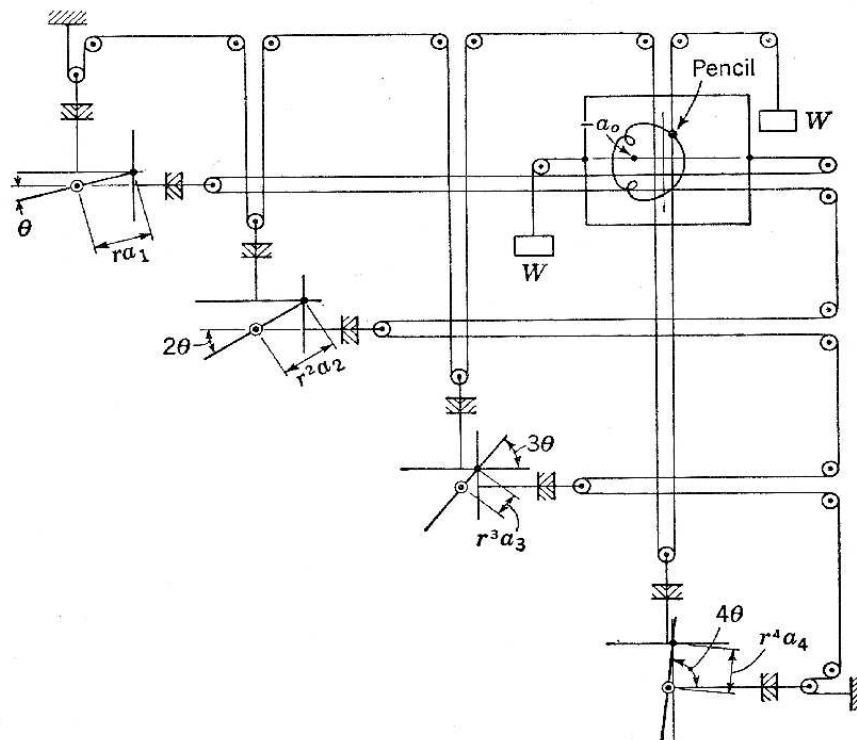
The harmonic synthesizer

Even the generation of trigonometric functions is quite simple using mechanical devices as the following picture shows (see [5][p.242]):
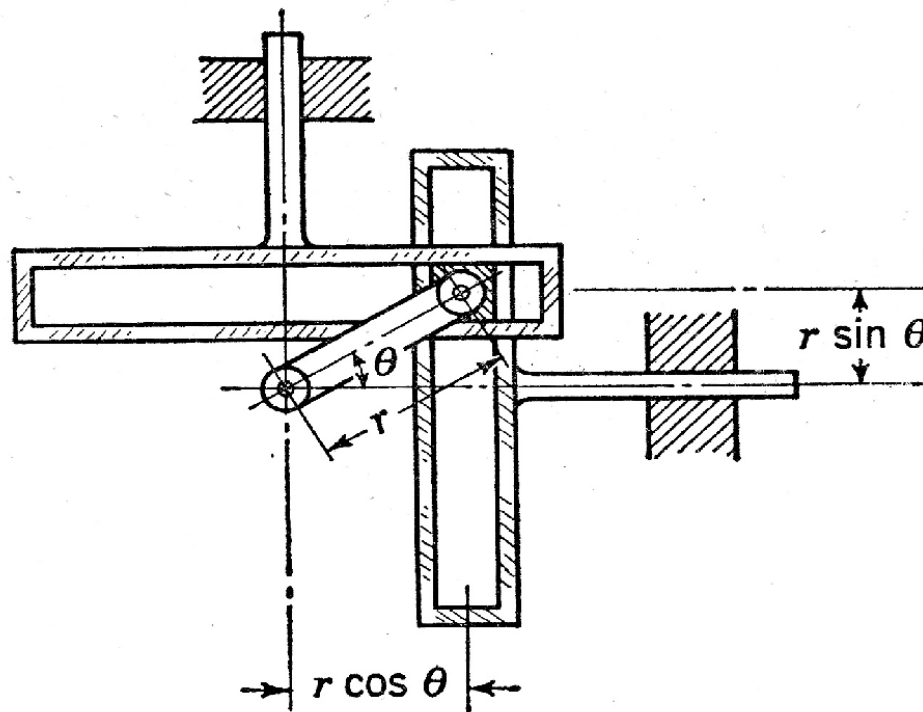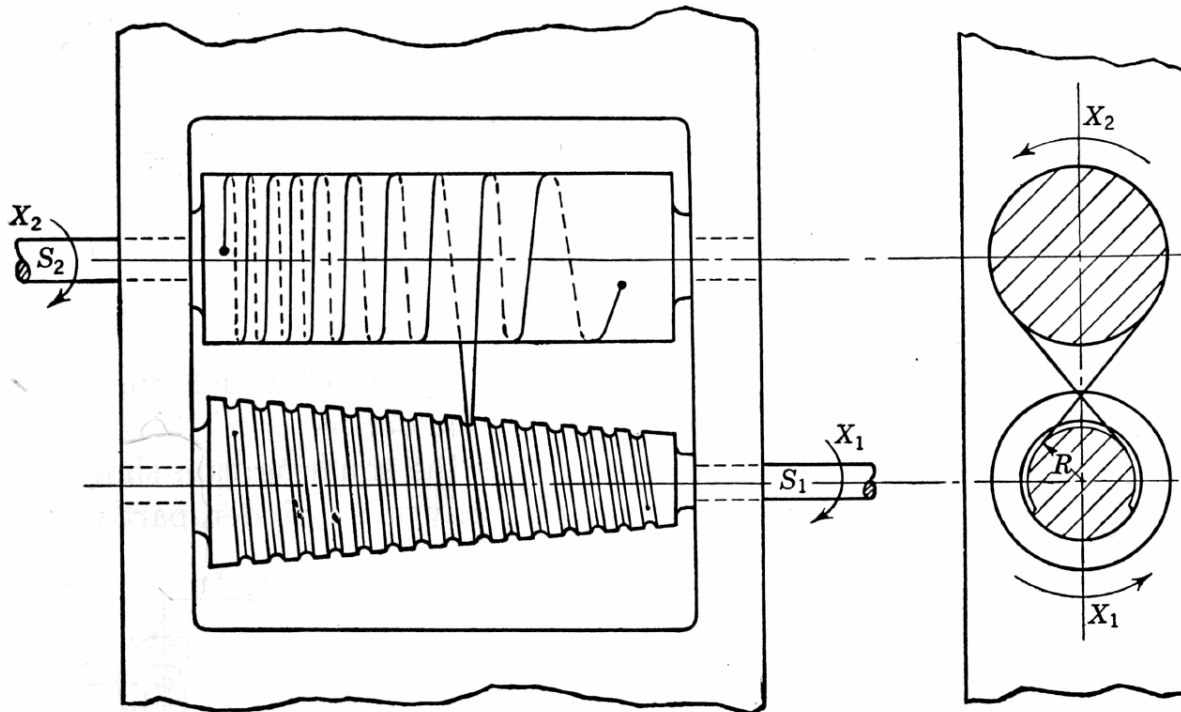
The harmonic synthesizer

Using mechanical adders a so called "harmonic synthesizer" can be built (cf. [5][p.242]):
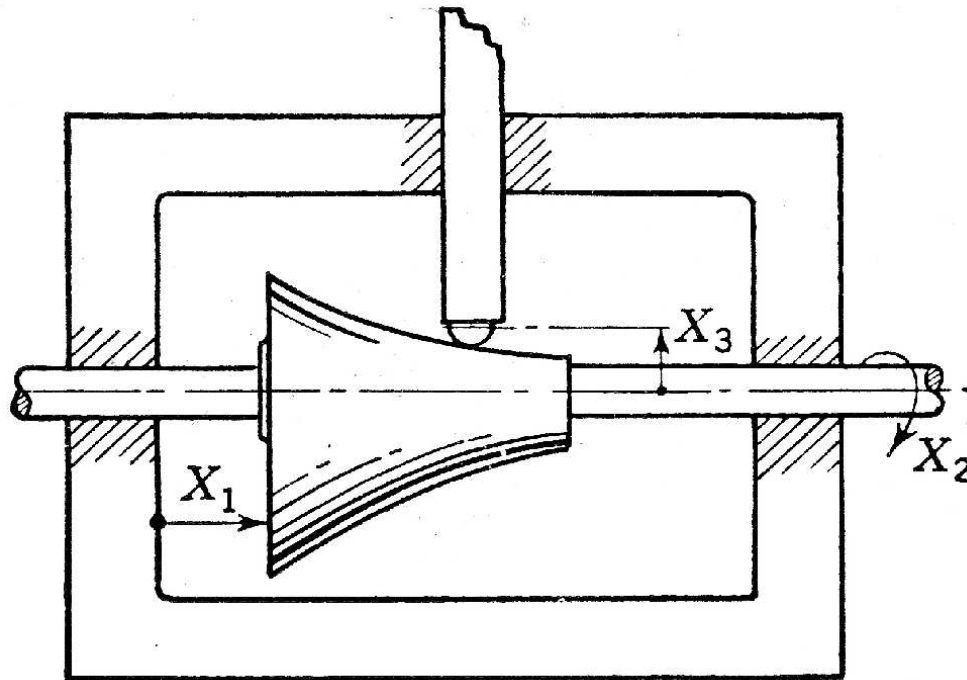
A squaring device

Special functions like $f(x) = x^2$ may be implemented as follows (see [9][p.22]):

Generating $f(x, y)$

Even more complex functions of two variables may be implemented as well (cf. [9][p.24]):

The end of the mechanical era

Mechanical differential analyzers (analog computers) had many disadvantages. This was the reason for their short life time:

- They were large, clumsy and difficult to maintain.

- The speed of calculations is severely limited by the moment of inertia of the components.

- The accuracy is limited to a few percent due to backlash, etc.

- The time required to setup the computer for a specific problem is very high because a lot of mechanical connections must be made.

Electronic analog computers

This led to the development of electronic analog computers which proved to be very influencial and had an incredibly high scientific value.

The first implementation of an electronic analog computer was done by Helmut Hoelzer in the late 1930s and early 1940s during his reserach for the V-weapons of the 3rd Reich (cf. [4][p.202ff]).

His main interest was in the field of simulation and control of propulsed rocket flight. Controlling a rocket like the V2 required a substantial amount of computational power which required some general approach like the electronic analog computer.

The key element of an all electronic analog computer is the so called *operational amplifier*.

The ideal operational amplifier

In the following an idealized operational amplifier with the following characteristics will be assumed:

- Infinite input impedance.

- Infinite gain.

- No drift.

- Differential inputs.
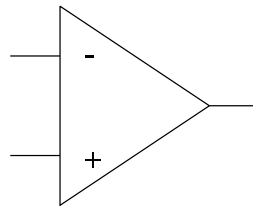
A real operational amplifier

All real implementations of operational amplifiers will suffer from drift, non-zero input impedance and – most important – non-infinite gain. A gain less than ∞ will yield (static) errors in computations so an amplifier suitable for an electronic analog computer will need a very large gain.
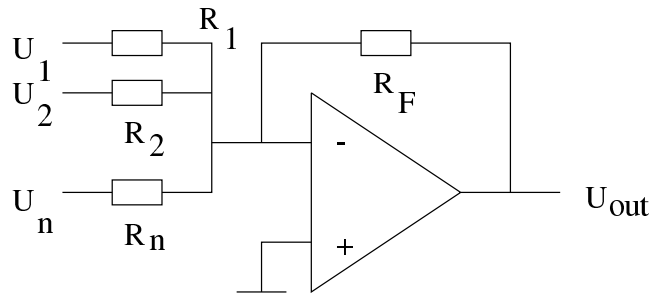
The following picture shows a tube based dual amplifier as used in the Solartron Analogue Tutor.

Both amplifiers are chopper stabilized to minimize drift effects and have a DC gain of 10^9! A most respective value even for today's standards.

A real tube based operational amplifier
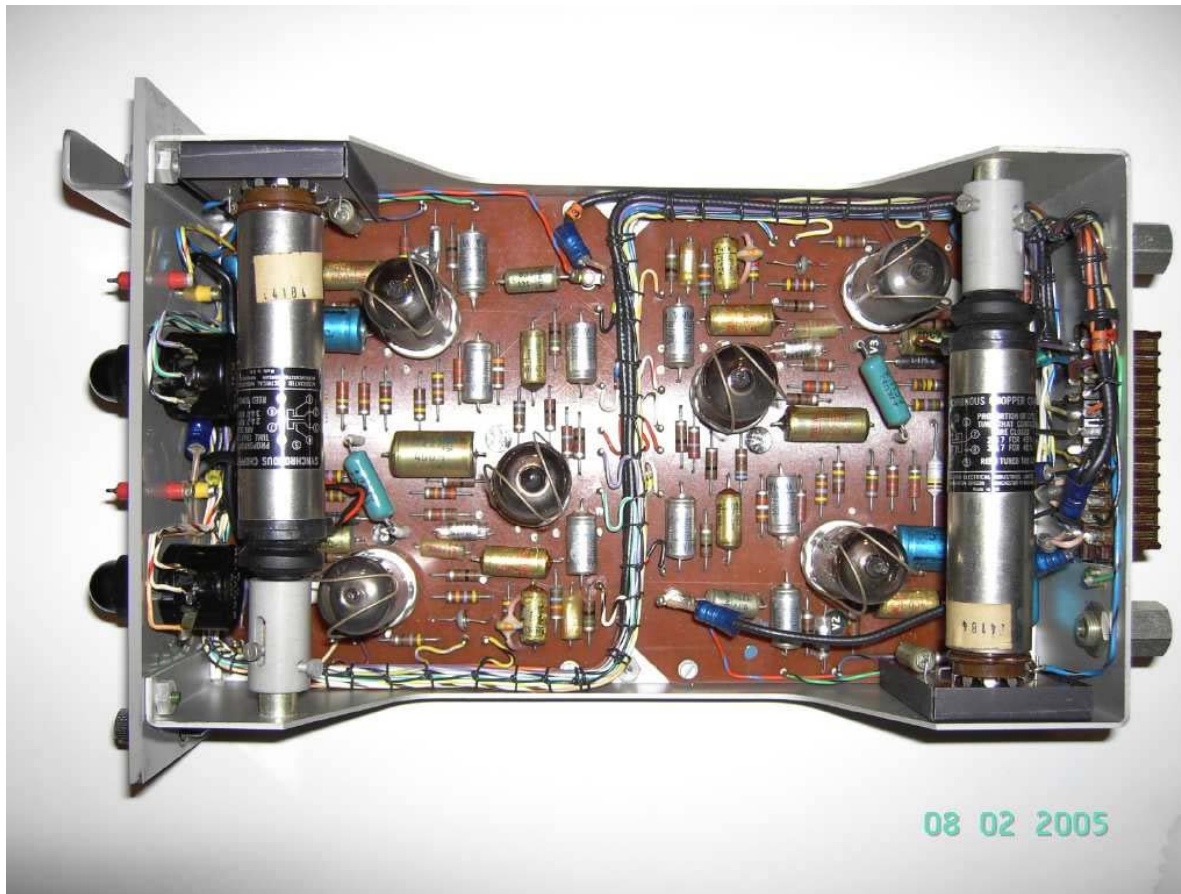
Summing

The sum of the currents at the summing junction has to be 0 due to the assumed gain $G = \infty$:

$$\sum_{i=1}^{n} \frac{U_i}{R_i} = -\frac{U_{\text{out}}}{R_F}$$

If $a_i = \frac{R_F}{R_i}$ then

$$-U_{\text{out}} = \sum_{i=1}^{n} a_i U_i$$

holds. Please note that a summer always implies a change of sign of the result!

Coefficient potentiometers

Nearly every calculation needs some way to multiply a value by a fixed amount.

In an electronic analog computer this is normally done with a precision 10-turn potentiometer connected as a voltage divider. This allows multiplication of an input value with a factor $1 \leq a \leq 1$.

The circuit of such a coefficient potentiometer and its graphic representation (a circle in which normally the multiplier is shown) can be seen in the figure below:
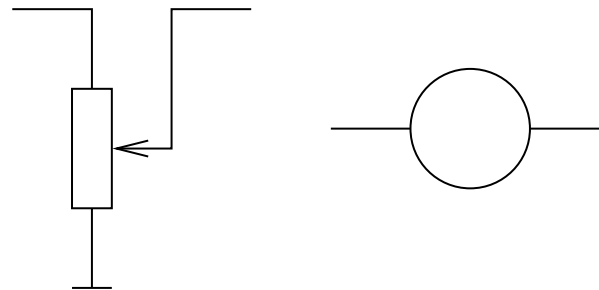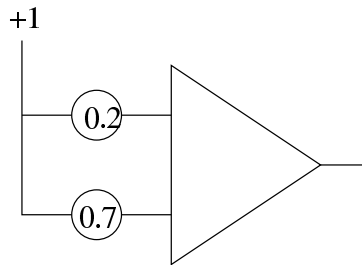
Coefficient potentiometers

The following picture shows the coefficient potentiometer drawer of the Telefunken RA742:
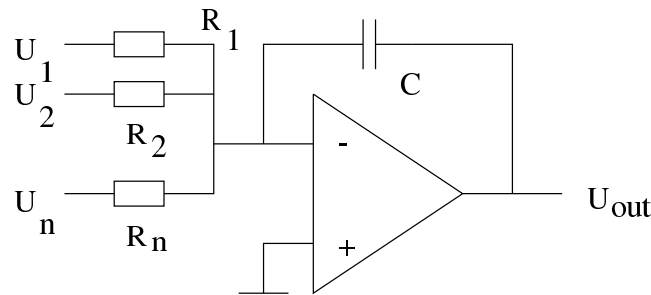
A summing example

To compute the sum $-(7 + 2)$ one would setup the analog computer as follows:

The triangle symbol denotes a summer as shown on the slide before, not a simple operational amplifier!

The circles denote two coefficient potentiometers.

Integrating

Integration is just as easy as summing – the main difference between a summer and an integrator is that the latter uses a capacitor as its feedback impedance as shown above. The output voltage of the integrator shown above is

$$-U_{\text{out}}(t) = U_0 + \int\limits_0^t \sum_{i=1}^n \frac{1}{R_i C} U_i \ \mathsf{d}t$$

where U_0 is the so called *initial condition*. Just like a simple summer an integrator changes the sign of its output.

Integrator control

The integrator is not only at the heart of every analog computer, it is by far the most complicated device since its behaviour does not only depend on its (analog) inputs.

An integrator can be run in ony out of three modes:

Mode	Description
Initial condition	In this mode the input summing network is disconnected from the amplifier while another input path is connected to the integration capacitor to charge it to the desired initial value.
Compute	The integrator calculates the time integral.
Halt	The integrator stops the calculation and holds the current value.

Run modes

An electronic analog computer normally features several run modes with respect to the control of its integrators:

Run mode	Description
Initial condition	All integrators are switched to IC.
Run	All integrators are placed in run mode.
Halt	All integrators enter the hold state.
Run with periodic halt	IC, Run, Hold, Run, Hold, Run, Hold, ...
Repetitive run	IC, Run, IC, Run, IC, Run, ...
Iterative run	The integrators are pooled into (at least) two groups: A normal group and a complementary group. While one group is in Hold the other one is in IC and vice versa.

What about deriving?

Obviously interchanging the feedback capacitor and the input resistor(s) would change the integrator into a differentiator which would seem like a good idea given the fact that differential equations rely on deriving instead of integrating.

Nevertheless analog computers normally make use of integrators only. This is a result of the fact that an integrator smoothes a signal thus suppressing noise, while a differentiator amplifies noise.

This requires one to get rid of all derivatives in a differential equation by integrating it a number of times before solving the equation(s) on an analog computer.
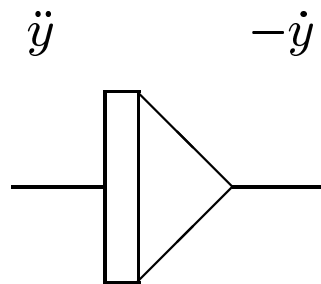
Solving a simple differential equation

Using summers and integrators it is now possible to solve simple differential
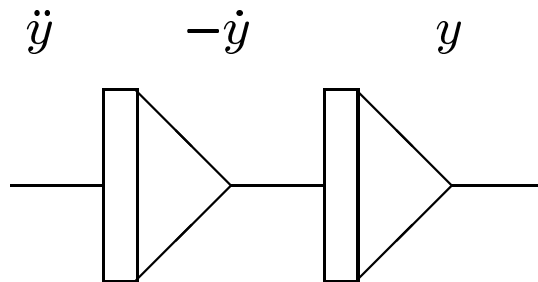equations like

$$\ddot{y} = -y.$$

First let us assume that $\ddot{y}$ is known...

Solving a simple differential equation

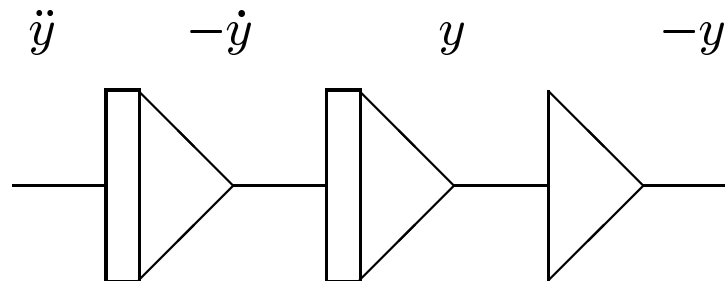Using $\ddot{y}$ as the input to an integrator we get the negative of its integral:

$$\ddot{y} \qquad\qquad -\dot{y}$$

Applying another integrator yields something quite familiar:

$$\ddot{y} \qquad -\dot{y} \qquad y$$

Solving a simple differential equation

Using a summer with one input the sign of the result can be changed easily:

$$\ddot{y} \qquad -\dot{y} \qquad\quad y \qquad\quad -y$$

Obviously the left hand side has to equal the right hand side according to the original differential equation, so connecting both sides realizes the equation on the analog computer yielding $\sin(\omega t)$ as the output of the computer circuit (U_0 denotes the initial condition, usually 1 in this case):
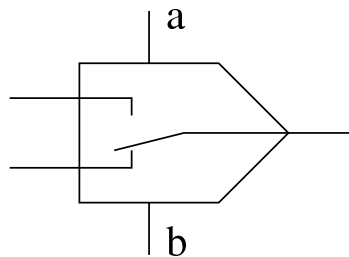
Solving a simple differential equation

$$\ddot{y} \qquad -\dot{y} \qquad y \qquad -y$$

$$\sin(\omega t)$$

$$U_0$$
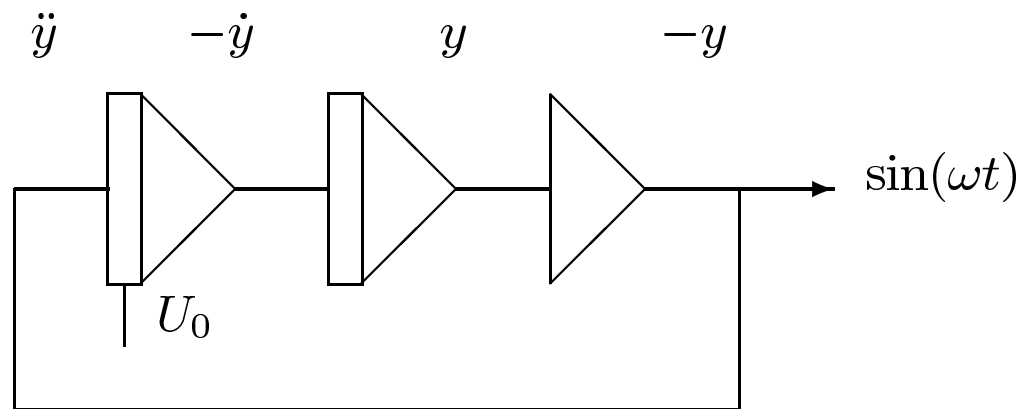
Comparators

If a high gain operational amplifier with a power output stage is used to drive a relay (or better an electronic switch), the resulting device is normally called a *comparator*. Its symbol is shown below:

Such a comparator normally has two control inputs a and b. It calculates the sum $a + b$. If this sum is greater than zero, the relay will be placed in the upper position, otherwise it will be in the lower position.

Using a comparator makes it possible to change the computing circuit during a calculation depending on external events such as reaching a boundary, etc.

Multiplication

Apart from summers, integrators and coefficient potentiometers, most relevant differential equations require the use of multipliers to solve them.

Multiplication is by no means simple using analog electronic circuits only, and a number of different approaches have been implemented:

- Servo multipliers

- Electron beam multipliers

- Hyperbolic field multipliers

- Time division multipliers

- Parabola multipliers

- Flux compensation multipliers (just kidding).

Servo multipliers

The earliest scheme adopted to perform electromechanic multiplication is the so called *servo multiplier*. It is a closed loop servo mechanism with a motor driven feedback potentiometer.

The position of the slider of this feedback potentiometer is controlled by an input signal, the multiplier. An operational amplifier with a power output stage capable of driving the servo motor will always set the angular position of the potentiometer in such a way that it corresponds to the input signal.

Some additional potentiometers are mounted on the same shaft as the feedback potentiometer. These potentiometers are used to perform multiple multiplications of different multiplicands by a single (and common) multiplier (which is the input signal to the servo loop).
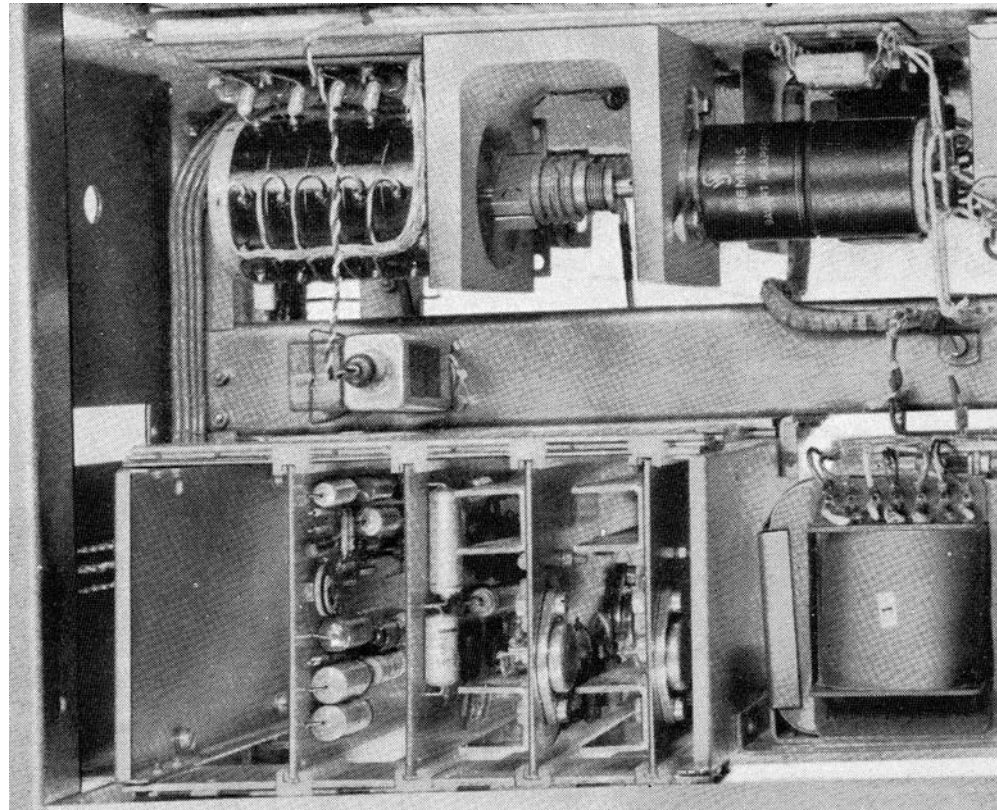
Servo multipliers

The following picture shows the simplified schematics of a servo multiplier (see [7][p.260]):
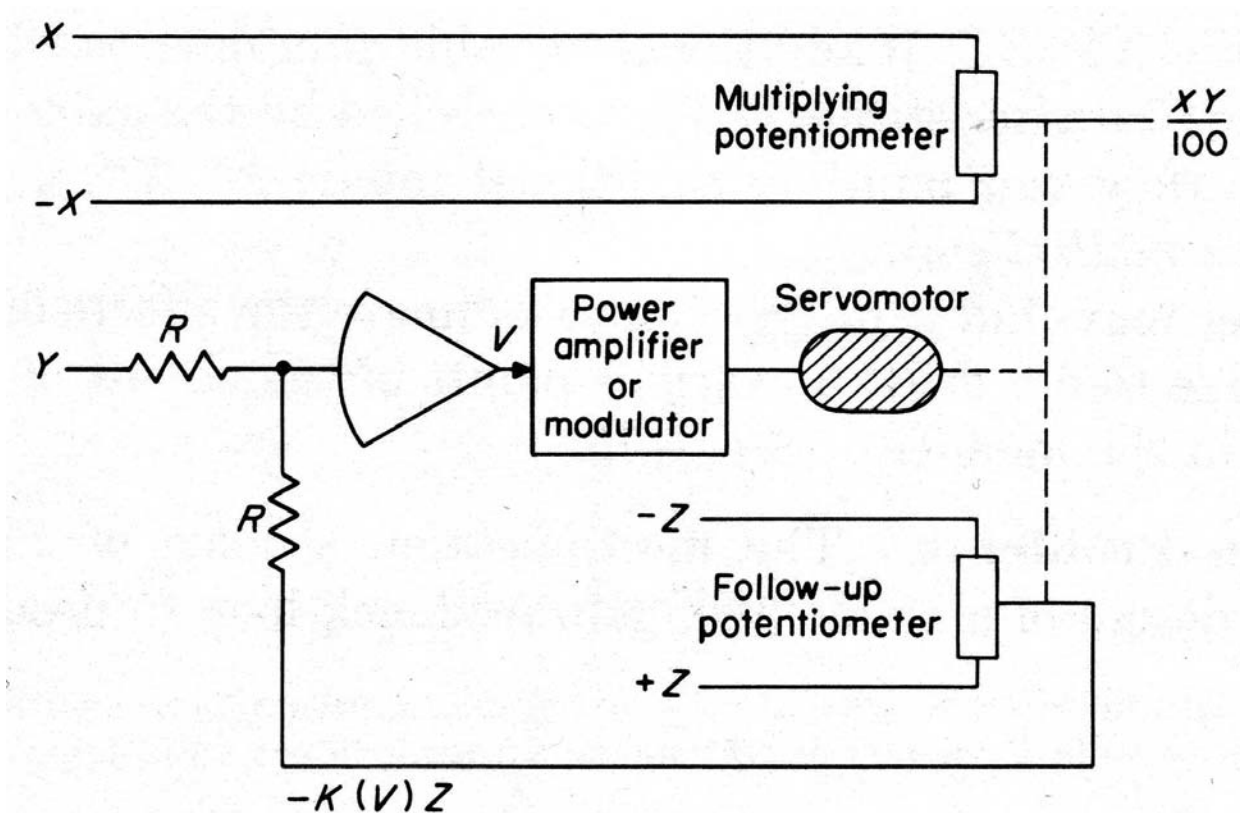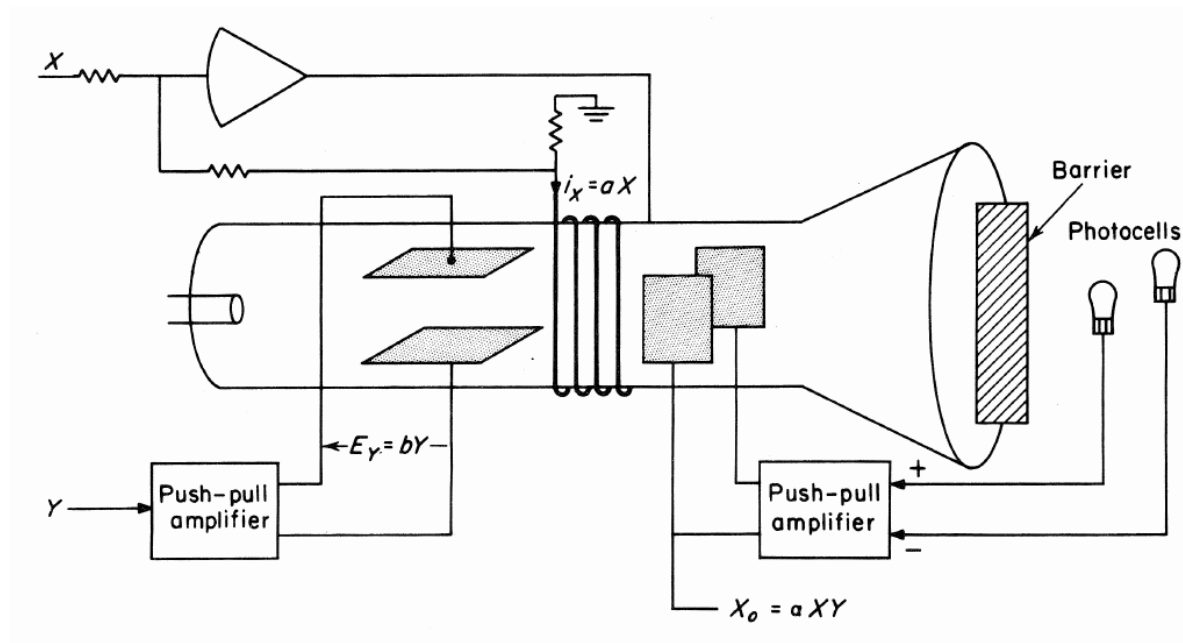
Servo multipliers

An actual servo multiplier as used in Telefunken's famous RA800 looks like this (see [2][p.83]):

Electron beam multipliers

Another multiplication technique makes use of electrostatic and electromagnetic deflection of an electron beam in an oscilloscope tube. The bandwidth of a device like this is limited by the magnetic deflection of the beam only.

The following picture shows an electron beam multiplier (cf. [7][p.294]):

Hyperbolic field multipliers

To overcome this bandwidth limitation, the hyperbolic field multiplier was developed in which only electrostatic deflection is used. Unfortunately this requires complex formed deflection plates and limits the obtainable precision of the device (see [1][p.17]):
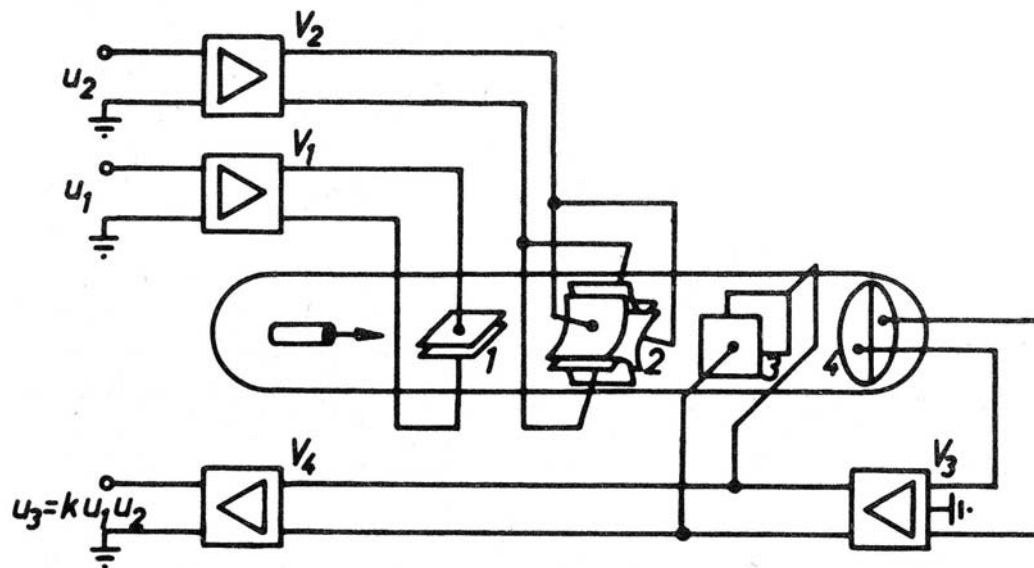
Time division multipliers

A time division multiplier allows the multiplication of a number of multiplicands by one common multiplier just as the servo multiplier does.

The multiplier is used to pulse modulate a square wave signal while each multiplicand is used to perform an amplitude modulation of this signal. The desired result is given by the area of the pulses, determined by their height and width.

The output signal is obtained by applying a low pass filter which creates an upper limit for the bandwidth of this computing device.

Since a number of multiplicands can be multiplied with a common multiplier, the time division multiplier is a perfect substitute for servo multipliers. Compared with these its bandwidth is considerably larger.

Time division multipliers

The following picture shows such a time division multiplier (see [7][p.269]):
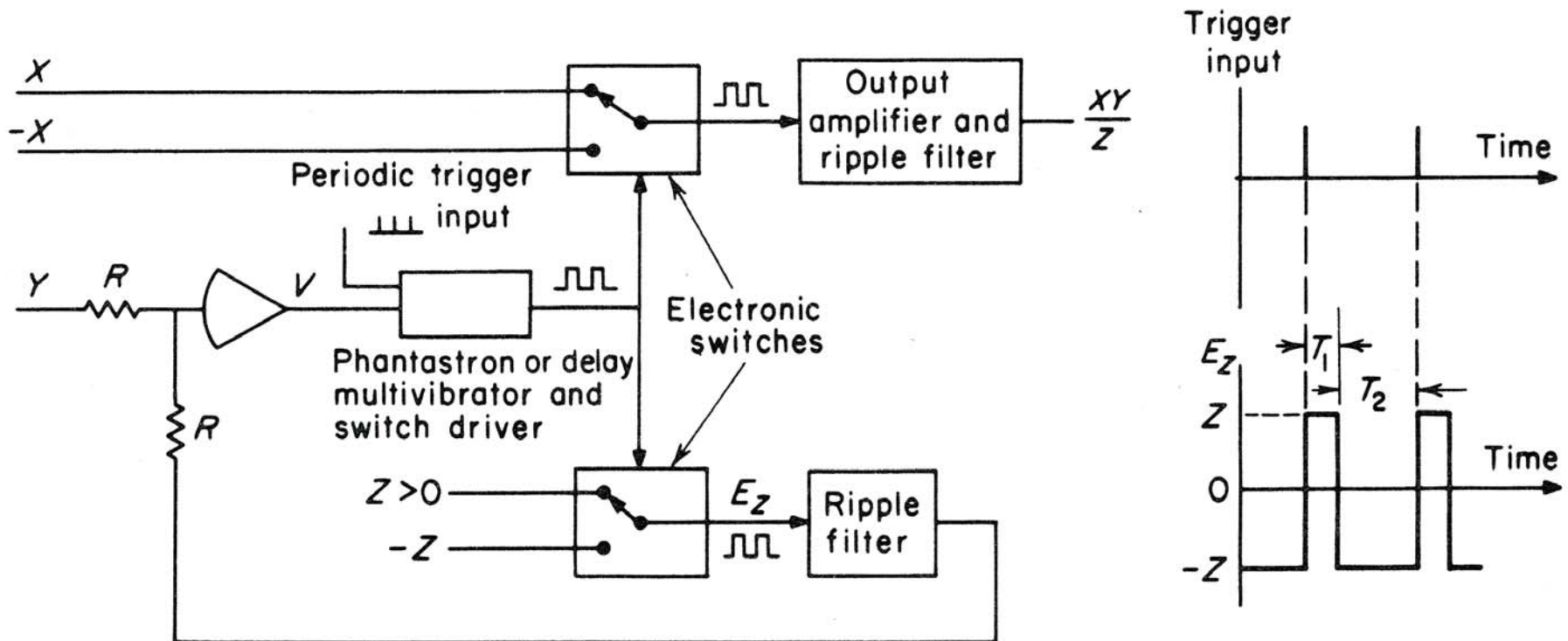
Parabola multipliers

A completely different multiplication technique is based on the fact that

$$\frac{1}{4}(x + y)^2 - (x - y)^2 = xy$$

The generation of the two square functions is performed by diode function generators which approximate a function by polygons. This limits the obtainable precision of a parabola multiplier to some 10^{-4} which is enough for nearly all applications.

The main advantage of such a multiplier is its very large bandwidth and its simple implementation. It requires only some biased diodes and an operational amplifier. A disadvantage is that both variables to be multiplied have to be available with both signs!

Parabola multipliers

The following picture shows the schematics of such a parabola multiplier as it is used in most transistorized analog computers (see [10][p.26]):
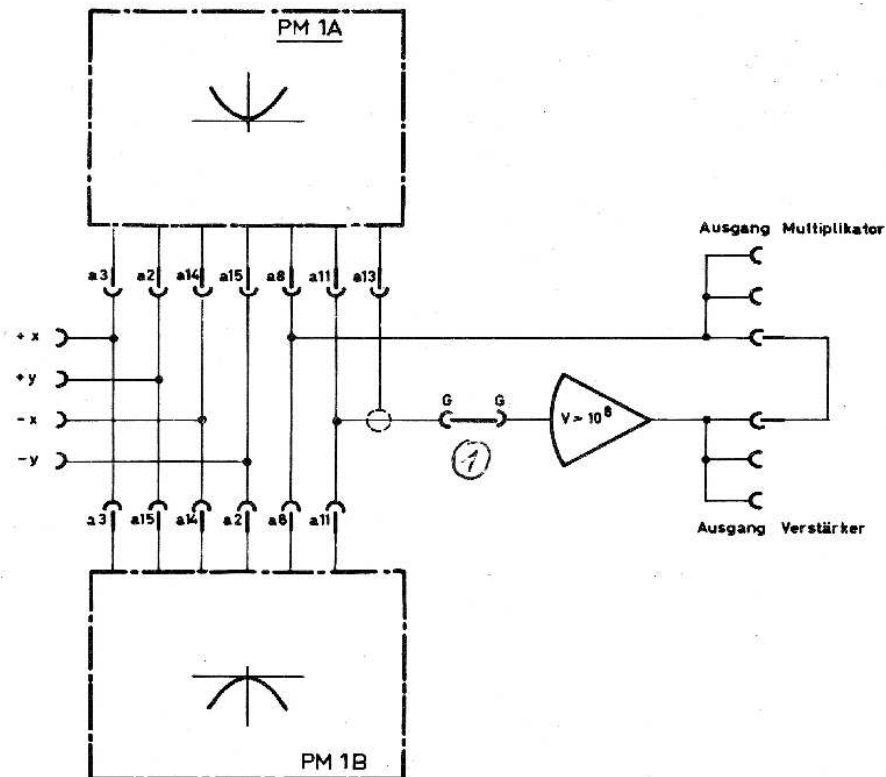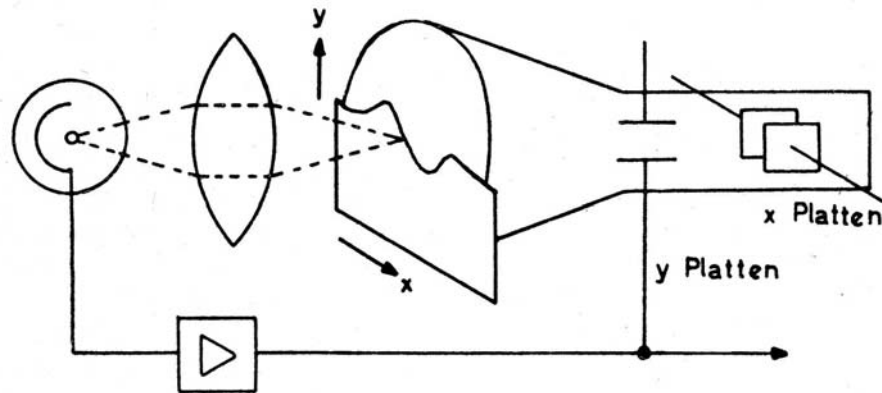
Function generators

Generating arbitrary functions is of prime importance for analog computers. During the years a wide variety of different function generators have been developed.

These function generators may be grouped like this:

- Function generators based on electromechanical devices like tapped potentiometers in a servo feedback loop (quite like a servo multiplier).

- Function generators based on special physical properties of some substances (voltage dependent resistors, temperature dependent resistors, special semiconductors, etc.).

- Function generators making use of a cathode ray tube in the feedback loop of an operational amplifier.

- Diode function generators which approximate (nearly) arbitrary functions by polygons. These are implemented using biased diodes.

Curve tracing function generators

The following picture (see [3][p.A3]) shows an oscilloscope tube based curve
following function generator:

The input to the X-deflection plates determines the position at which the function
value has to be determined. The function is implemented as an opaque mask placed
in front of the tube screen. A feedback loop will generate an Y-deflection voltage in
such a way that the beam will just be on top of the mask, thus following the function.

Diode based function generators

Diode based function generators approximate (more or less) arbitrary functions by polygons as shown in the picture below:
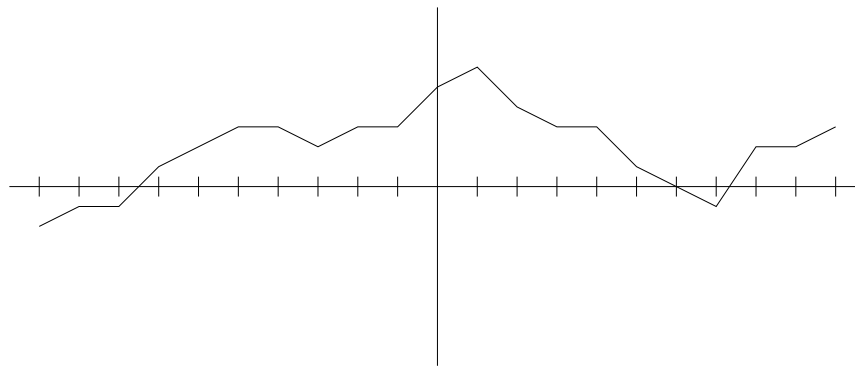

Most implementations have either fixed sampling points (in this case there are normally 21 of these points spaced $\frac{1}{10}E$ apart where E denotes the machine unit (either 10 V or 100 V)) or variable sampling points.

Each sampling point corresponds to a diode biased in a way that it will begin to conduct when the x-input reaches this value. By means of a potentiometer the slope added to the polygon by this diode can be set.

Diode based function generators

The following picture shows the wiring diagram of a diode based function generator allowing positive and negative slopes for the partial polygons (cf. [10][p.28]):
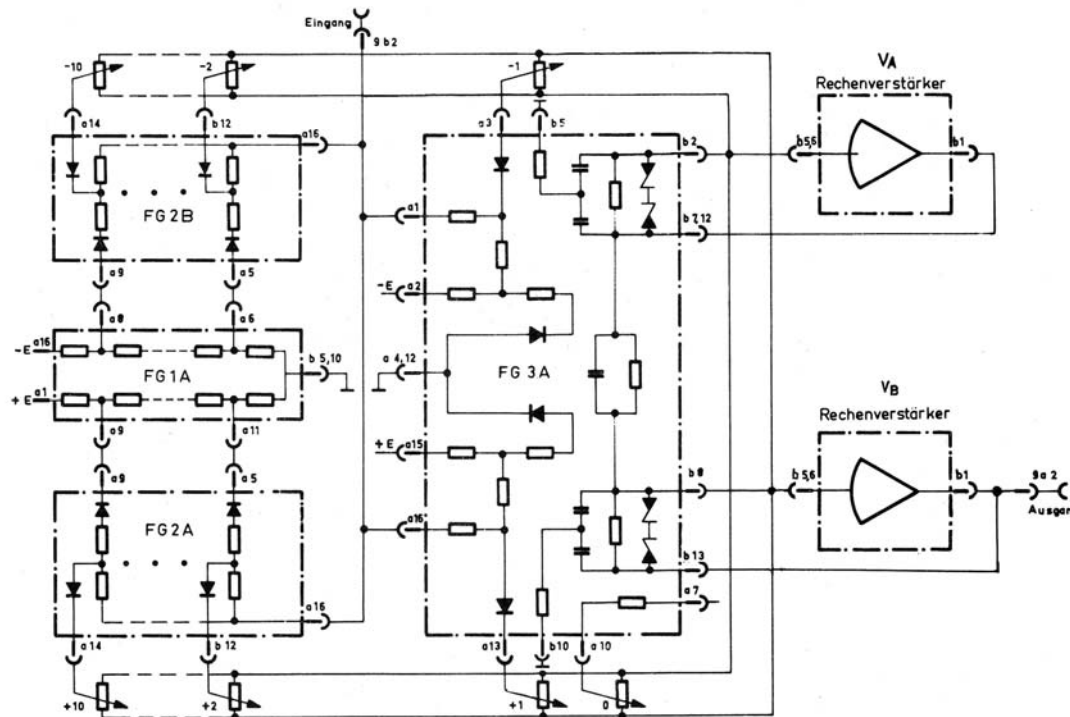
The diode function generator of the RAT700
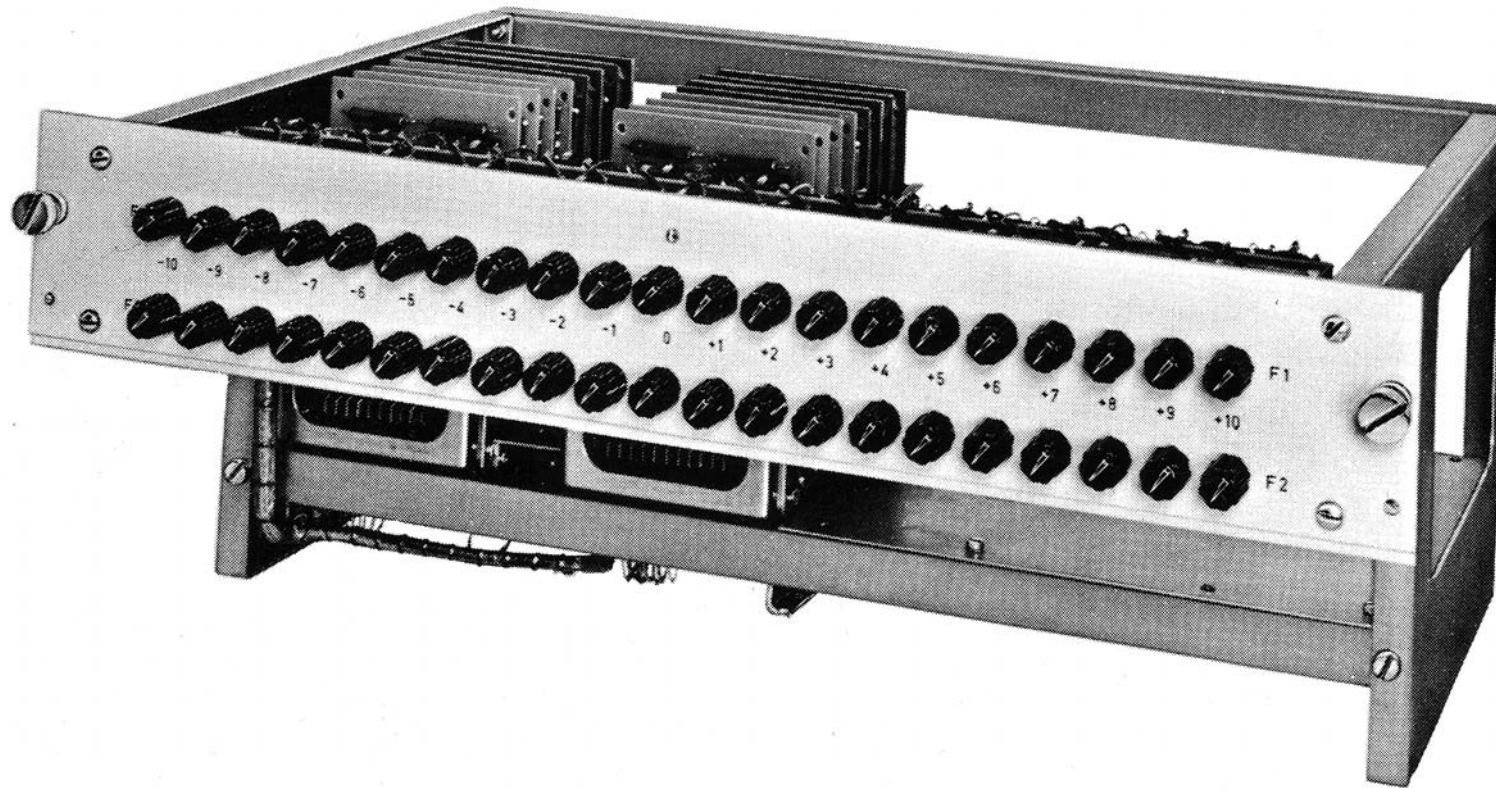
Generating inverse functions

Generating the inverse $f^{-1}(x)$ of a given function $f(x)$ is quite simple by placing a function generator set up to generate $f(x)$ in the feedback loop of an operational amplifier:
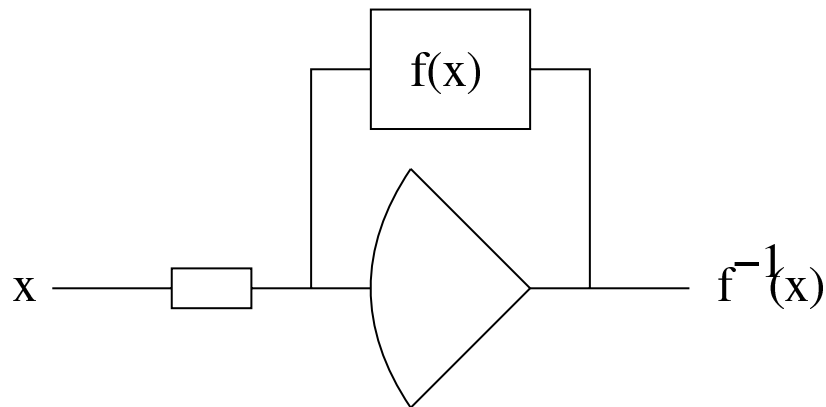
This circuit is normally employed to generate a square root from a square, an exponential function from a logarithmic function, etc.

A practical implementation will normally need a small (some pF) capacitor accross the function generator to enhance the stability of the circuit.

Symbols used in analog computing

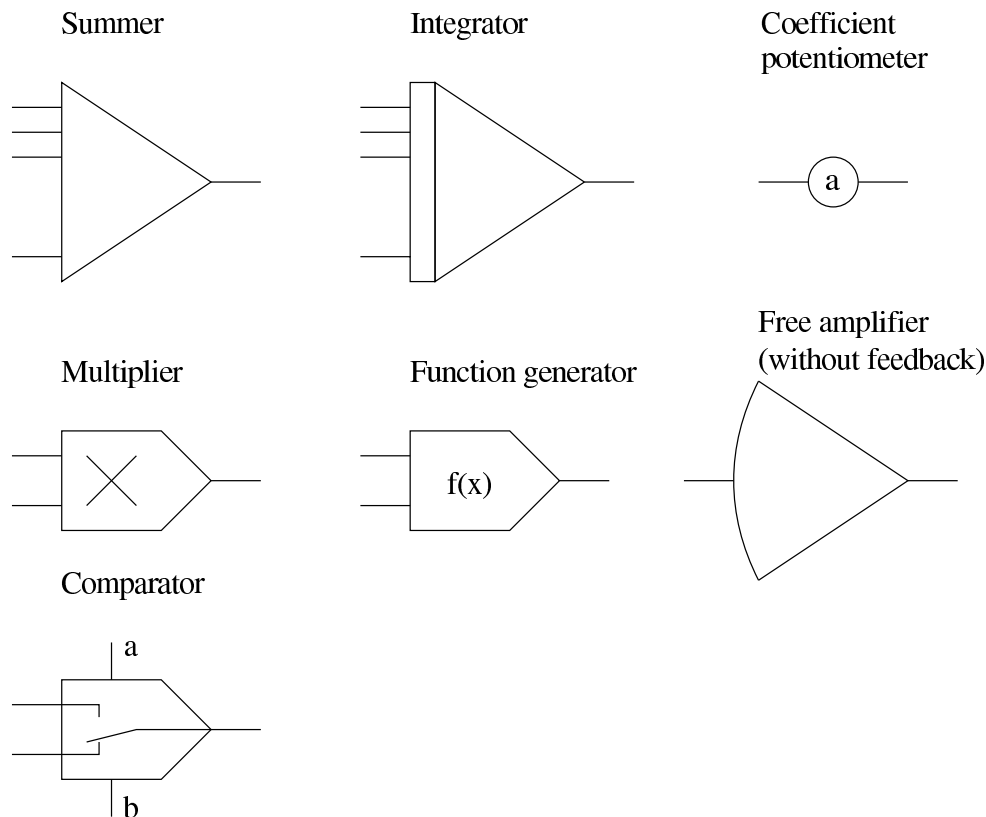The following symbols are normally used in analog computing to denote the various
computer elements used in solving a given problem:

Summer   Integrator   Coefficient
                                                      potentiometer

Multiplier   Function generator   Free amplifier
                                                      (without feedback)

Comparator

Patch fields

All inputs and output of the elements of an electronic analog computer are normally brought together at the so called *patch panel*. These patch panels are normally removable to allow changing a program easily. The following picture shows the patch field drawer of the Telefunken RA742:
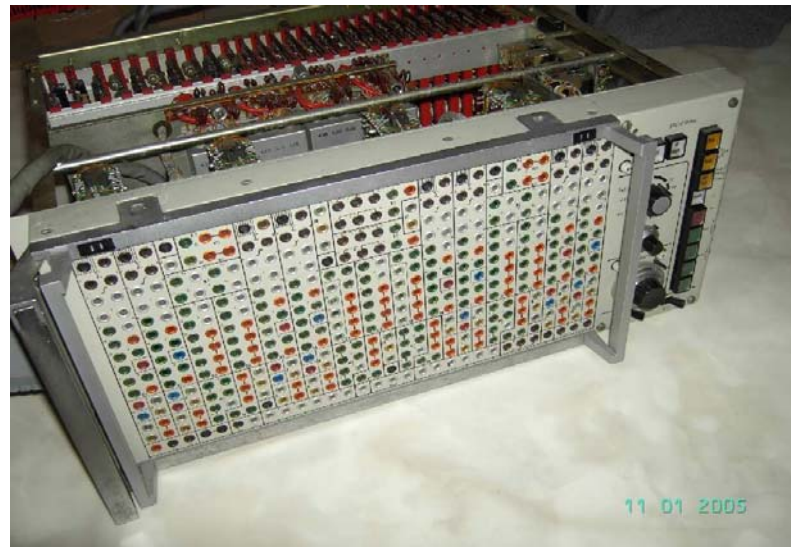
Setup and control

Setting up an electronic analog computer and solving a given problem requires (at least) the following steps:

- Setup of the circuit on the patch panel.

- Setting all coefficient potentiometers.

- Setup of all used (diode) function generators.

- Selecting the desired run mode and setting the timers.

Some analog computers facilitate these setup procedures by supporting automatic potentiometer setting (either by servo circuits or by employing digital potentiometers) as well as automatic function generator setup.

How does an analog computer look like?

The Telefunken RA740

The system shown before is the Telefunken RA740 table top analog computer (it is difficult to find a table for it since its weight exceeds 100 kg!). It consists of the following parts (from top to bottom):

- Power supply and 15 operational amplifiers – these are located in the upper drawer.

- Two diode function generators – the potentiometers are normally behind a lid which is missing here.

- 19 coefficient potentiometers and a 11 position fixed voltage divider switch.

- The patch panel – this drawer contains the compute networks as well as the control panel with the timing generator, etc.

Towards hybrid computing

When only simple differential equations are to be solved, an electronic analog computer has a lot of advantages over a digital machine.

Unfortunately many real world problems have quite special requirements like iterative computation or free control of integrators, etc.

An example for this are optimization procedures. Here several groups of integrators are used – one group calculates the initial values for the next group, etc. During the calculation several events (crossing limits, etc.) will be used to trigger other events like placing one integrator group into hold mode while the next group will pick up this value, and so on.

Early attempts

In the mid 1960ies Telefunken introduced a digital attachement, called "DEX" for "Digitalzusatz". The DEX was a table top device with a removable patch board just like an analog computer but it contained logic gates, flipflops, delay lines, etc. With the aid of the DEX it was possible to implement complicated iterative calculation schemes on a Telefunken table top analog computer. The picture below shows a $0 \ldots 5$ counter patched on the DEX:
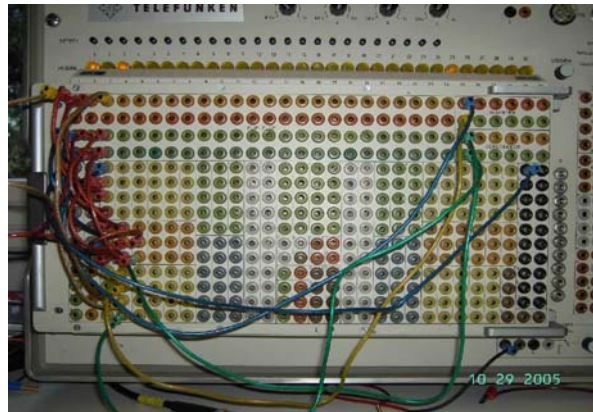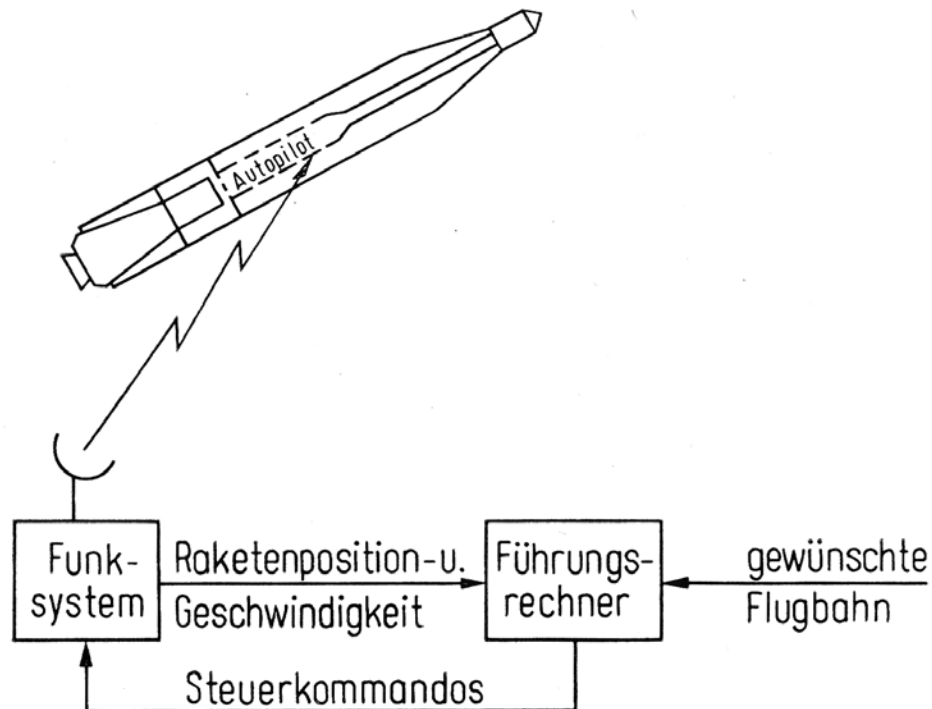
RA742 with DEX102
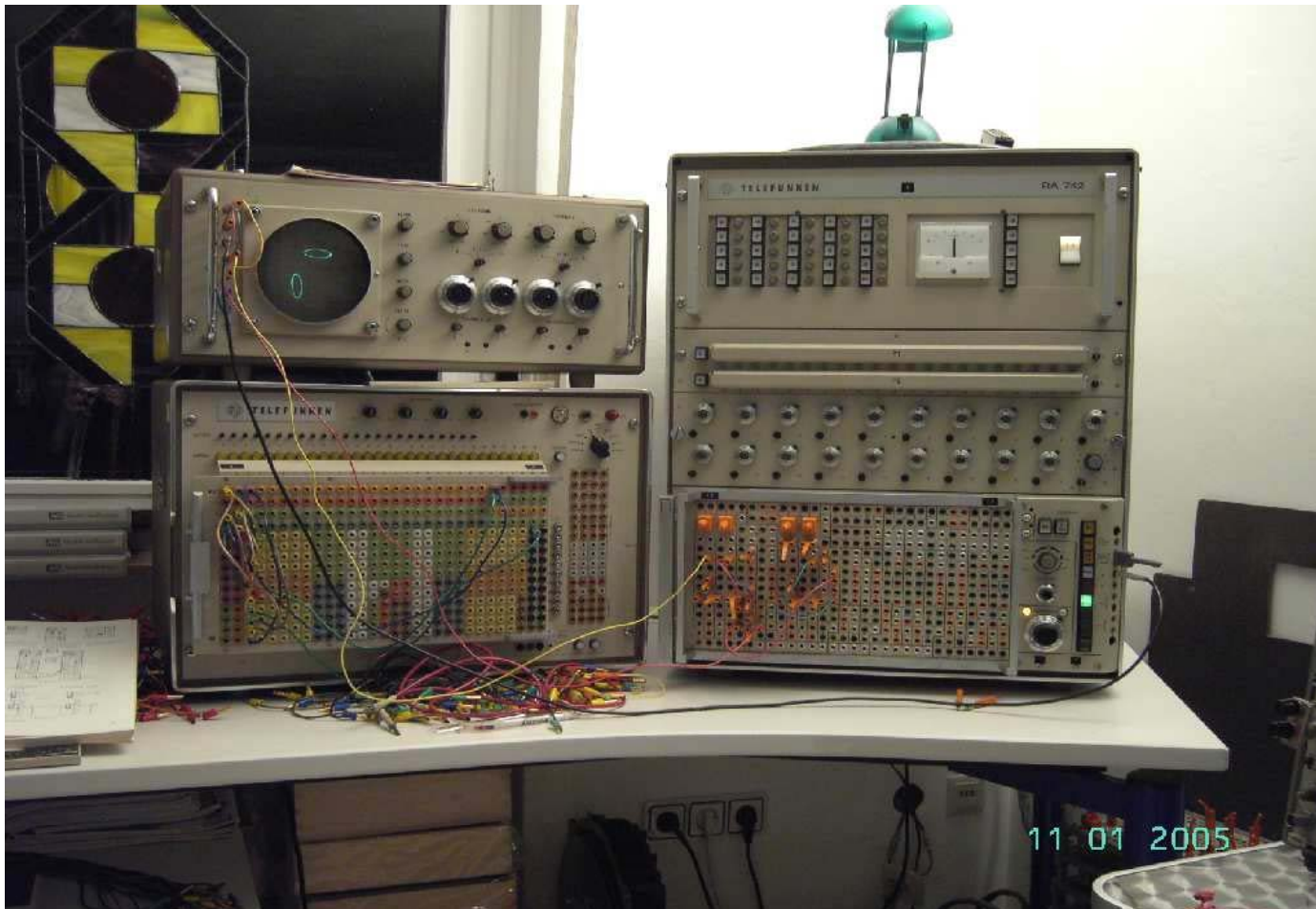
Simulating a remotely controlled rocket
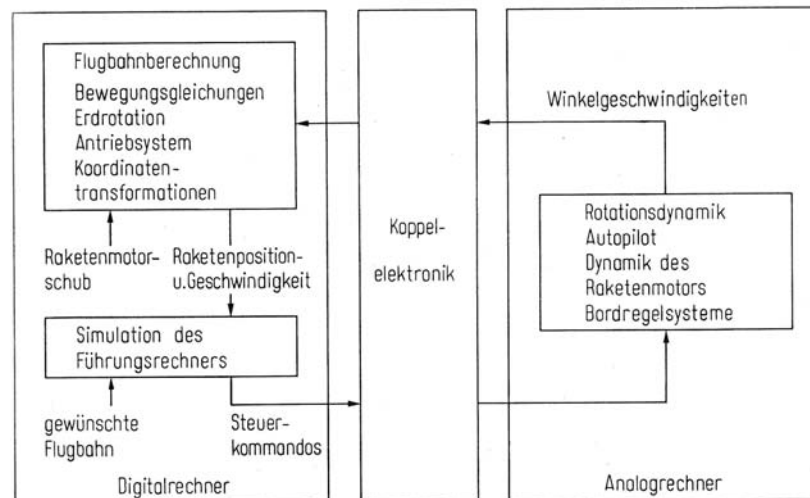
To simulate a remote controlled rocket as shown in the following picture, a simple analog computer may not be sufficient (cf. [6][p.84]):

Simulating a remotely controlled rocket

A so called "hybrid computer" – a combination of a digital computer and an analog computer – has a lot of advantages in simulations like this. A typical setup is shown in the picture below (see [6][p.85]):

Simulating a remotely controlled rocket

In this example, the analog computer takes care of the time critial (real time, in fact) parts of the simulation such as the rocket and flight dynamics, simulation of the internal systems of the rocket and the like.

The digital computer on the other hand will simulate the ground equipment, the remote control system. It supplies the analog computer with input values by means of analog digital converters while it receives data from the analog computers via digital analog converters and/or comparators.

Thus each system is employed in such a way to make maximum use of its specific characteristics. The analog computer trumps the digital machine with respect to sheer speed of calculation while the digital computer has the advantage of arbitrary high precision, large memory, etc.

Example of a hybrid computer

The following picture shows the Telefunken hybrid computer system HRS 900 (cf.
[6][p.78]). It consists of a Telefunken digital processor 90-40, the hybrid interface
HKW 900 (containing the AD/DA-converters as well as the necessary interface
logic) and finally the precision analog computer RA770:
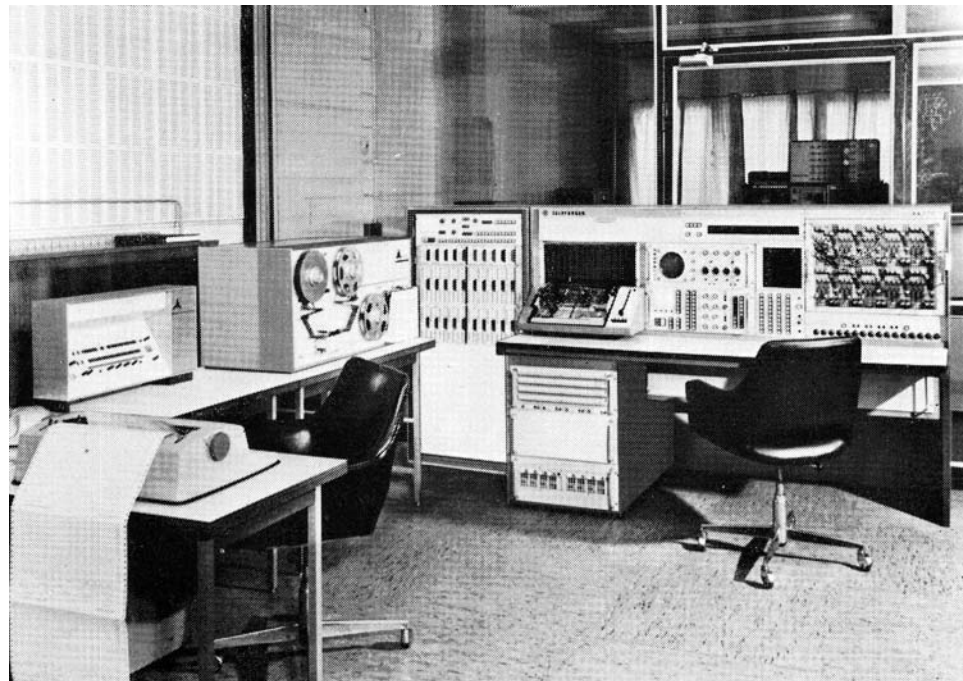
Areas of application

Systems like this were in widespread use in a variety of places, for example:

- Aeronautics and astronautics research at NASA (simulation of rockets, capsules, propulsion and navigation systems, etc.).

- Car simulations at Ford, GM, Opel, VW, etc.

- Research in pharmacy (pharmacokinetics, blood vessel simulation, etc.).

- Chemical research (reaction dynamics).

- Engineering sciences (simulation of complex mechanical equipment, plastodynamics, plant simulation, etc.).

- Nuclear physics (reactor simulation, fault simulation, . . .).

- Military research (simulation, etc.).

The last hybrid computers

One of the last hybrid computer systems ever built is the Dornier 960. Its analog

processing part consists of 16 drawers each containing a variety of summers,

integrators, etc. Its digital processing part consists of a CAMAC frame with either an

embedded microprocessor running a real time BASIC or with the possibility to

attach to a Data General NOVA processor.

Help

As you may have noticed, analog computing is my passion! Therefore I would like to ask you for help:

- I am trying to save analog computers from scrap where ever I can. If you happen to know about a system looking for a good home, please let me know. (I am not afraid of large systems and I would really love to get an RA770 or an RA800(H), for example! I will pay for all expenses to save the machine from scrap and I will arrange shipping, etc.)

- I am interested in trivia, documentation, computer setups, sales brochures, etc.

- Please spread the word and help to save these machines from getting lost and forgotten.

You can reach me always at `ulmann@vaxman.de` or by mobile phone at 0177/5633531 (in Germany) in case of an emergency. Thank you very much!

References

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